SOS3003
Applied data analysis for social science
Lecture note 07-2010

Erling Berge
Department of sociology and political science
NTNU

Literature

- Fitting Curves
  Hamilton Ch 5 p145-173
- Robust Regression
  Hamilton Ch 6 p183-212

Fitting Curves

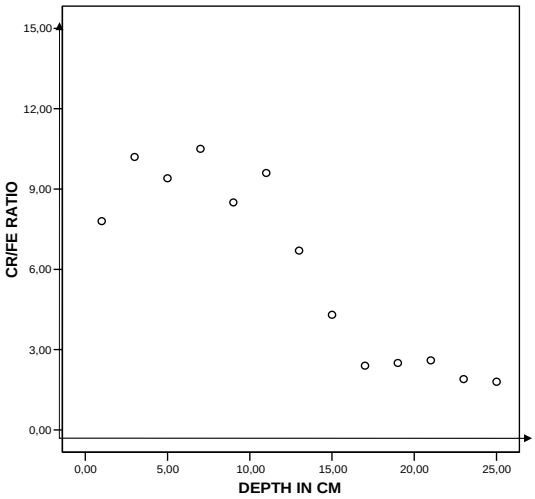
- A correctly specified model require that the function linking x-variables and y-variable is true to what really exist: is the relationship linear?
- Data can be inspected by means of band regression or smoothing
- The theory of causal impact can specify a non-linear relationship
- For phenomena that cannot be represented by a line we shall present some alternatives
  - Curvilinear regression
  - Non-linear regression

Band regression

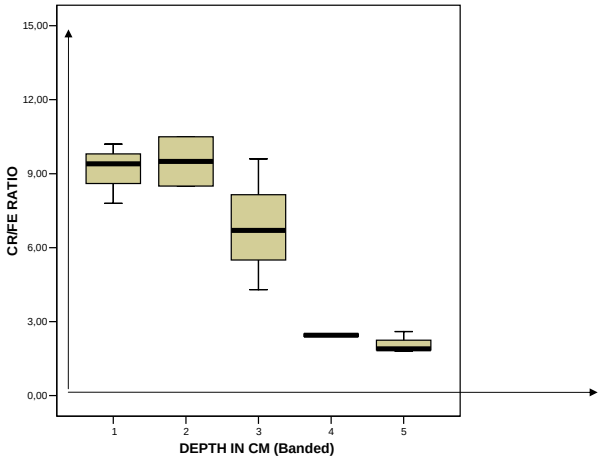
- Can be used to explore how the relationship among the variables actually appears
- If we can see a non-linear underlying trend of the data we must through transformations or use of curves find a form for the function better representing the relationship

Pollution at different depths in sediments outside the coast of NH

- Pollution measured by the ratio chromium/iron at different depths of various sediment samples
- Is the relationship linear?



Medians of 5 bands: rate of chromium/iron in sediments outside the coast of NH



The relationship is obviously non-linear

Transformed variables

- Using transformed variables makes a regression curvilinear. The transformation makes the original curve relationship into a linear relationship
- This is the most important reason for a transformation
- At the same time transformations may rectify several other types of statistical problems (outliers, heteroscedasticity, non-normal errors)
- Procedure:
  - Choose an appropriate transformation and make new transformed variables
  - Do a standard regression analysis with the transformed variables
  - To interpret the results one usually will have to transform back to the original measurement scale

The linear model

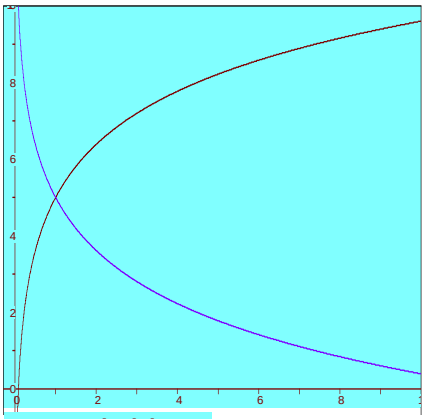
$$y_i = \beta_0 + \sum_{j=1}^{K-1} \beta_j X_{ji} + \varepsilon_i$$

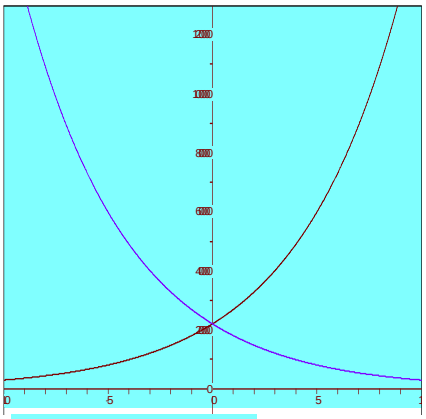
- In the linear model we can transform both x- and y- variables without any consequences for the properties of OLS estimates of the parameters
- OLS is a valid method as long as the model is linear in the parameters

Curvilinear Models

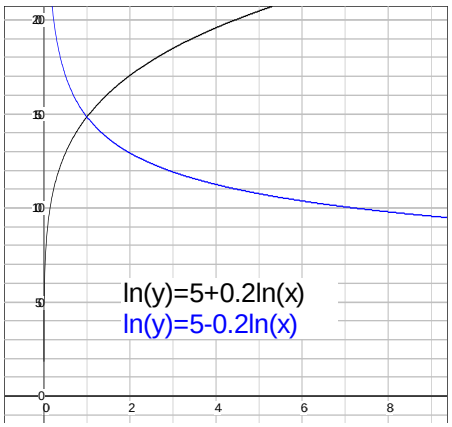
- Practically speaking this is regression with transformed variables
- We shall take a look at how different transformations provide different forms for the variable relations
  - Semi-logarithmic curves
  - Log-Log curves
  - Log-reciprocal curves
  - Polynomials (2 and 3 order)

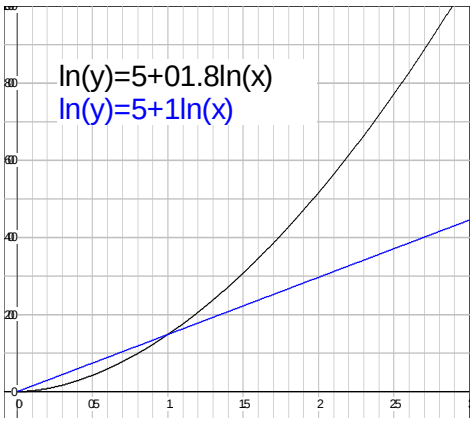
Semilog curves Fig 5.2 in Hamilton



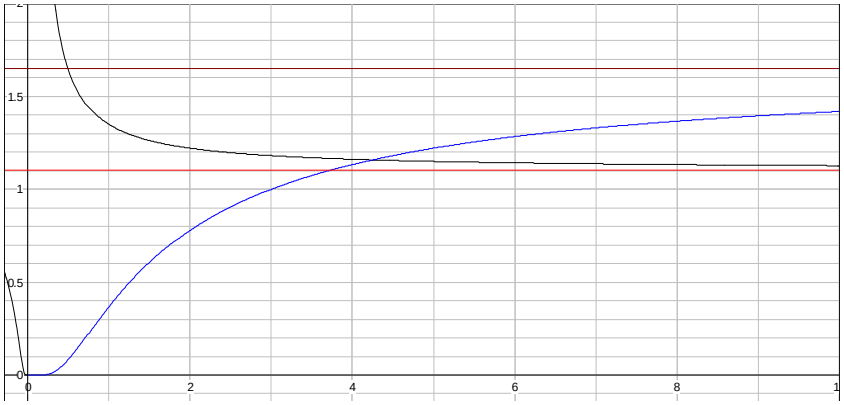


Log-log curves Fig 5.3 in Hamilton





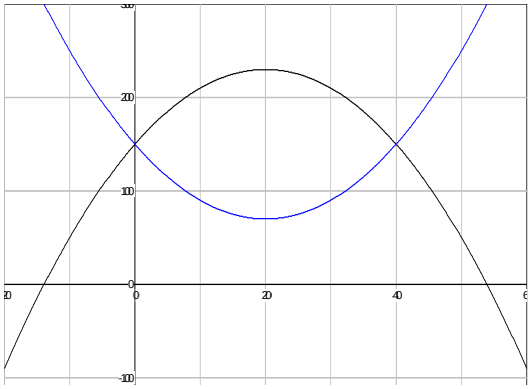
Log-reciprocal curves Fig 5.4 in Hamilton



ln(y)=0.1+0.2/x
ln(y)=0.5-1.5/x
Horizontal line through (0, 1.105)
Horizontal line through (0, 1.649)

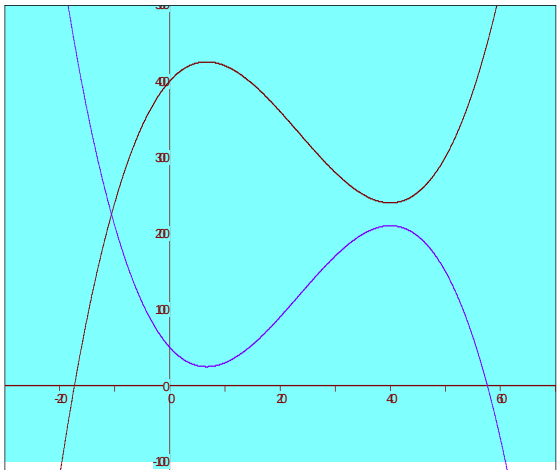
The horizontal lines give the value of y when x grows towards infinity: the asymptote for y

Second order polynomials Fig 5.5 in Hamilton



y=150+8x-0.2x^2
y=150-8x+0.2x^2

Third order polynomials Fig 5.6 in Hamilton



y=400+8x-0.7x^2+0.01x^3
y=50-8x+0.7x^2-0.01x^3

Choice of transformation

- Scatter plot or theory may provide advice
- Otherwise: transformation to symmetry gives the best option
- The regression reported in table 3.2 in Hamilton proved to be problematic
- Regression with transformed variables can reduce the problems

Choice of transformation in table 3.2 in Hamilton

Y = Water use 1981	$Y^*=Y^{0.3}$ provides approximate symmetry
X_1 = Income	$X_1^*= X_1^{0.3}$ provides approximate symmetry
X_2 = Water use 1980	$X_2^*= X_2^{0.3}$ provides approximate symmetry
X_3 = Education	Transformations are inappropriate
X_4 = Pensioner	Transformations do not work for dummies
X_5 = # people in 1981	$X_5^*= \ln(X_5)$ provides approximate symmetry
X_6 = Change in # people	$X_6 = X_5 – X_0$ (= # people in 1980)
X_7 = Relative change in #people	$X_7^* = \ln (X_5/X_0)$

Regression with transformed variables
Tab 5.2 in Hamilton

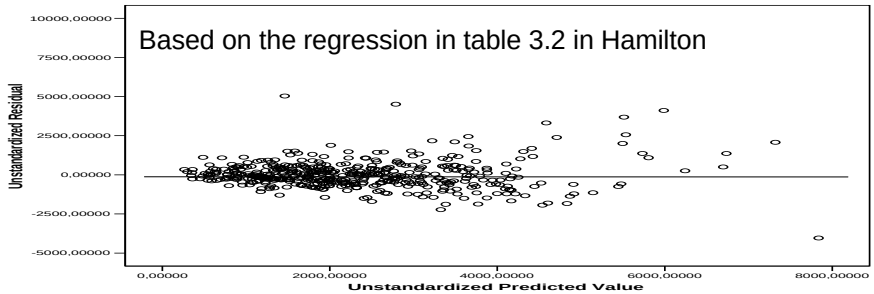
Dependent Variable: $(Wateruse81)^{0.3}$	B	Std. Err	t	Sig.
(Constant)	1,856	,385	4,822	,000
$Income^{0.3}$	,516	,130	3,976	,000
$Wateruse80^{0.3}$	,626	,029	21,508	,000
Education in Years	-,036	,016	-2,257	,024
Retired?	,101	,119	,852	,395
Ln(# of people81)	,715	,110	6,469	,000
Ln(people81/people80)	,916	,263	3,485	,001

Table 3.2 (Hamilton p74)

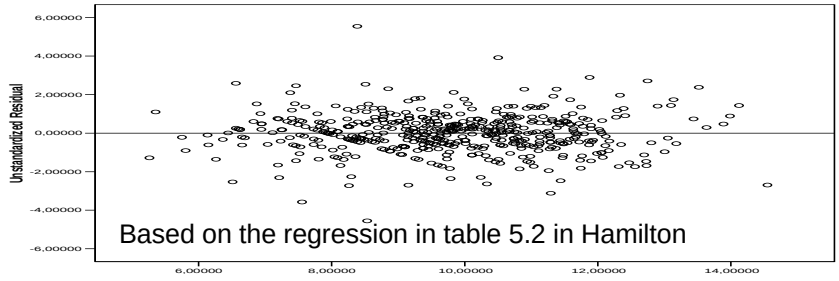
Dependent Variable: Summer 1981 Water Use	B	Std. Error	t	Sig.	Beta
(Constant)	242.220	206.864	1.171	.242	
Income in Thousands	20.967	3.464	6.053	.000	.184
Summer 1980 Water Use	.492	.026	18.671	.000	.584
Education in Years	-41.866	13.220	-3.167	.002	-.087
Head of house retired?	189.184	95.021	1.991	.047	.058
# of People Resident, 1981	248.197	28.725	8.641	.000	.277
Increase in # of People	96.454	80.519	1.198	.232	.031

How do we interpret the coefficient of ”Increase in # of People” ?

What leads to less water use after the crisis?



Residual against predicted Y



Other consequences of the transformations

- Two cases with large influence on the coefficient for income (large DFBTAS) do not have such influence (fig 4.11 and 5.9)
- One case with large influence on the coefficient for water use in 1980 do not have that large influence (fig 4.12 and 5.10)
- Transformation to symmetrical distributions will often solve many problems – but not always
- And it creates a new one: interpretation

Interpretation

- The model estimate now looks like this

$$y_i^{0.3} = 1.856 + 0.516x_{1i}^{0.3} + 0.626x_{2i}^{0.3} - 0.036x_{3i}$$

$$+0.101x_{4i} + 0.715\ln(x_{5i}) + 0.916\ln(\frac{x_{5i}}{x_{0i}})$$

- The interpretation of the coefficients are not so straightforward any more. For example: the measurement units of the parameters have been changed
- The simplest way of interpreting is to use conditional effect plots

Conditional effect plot

- Should be used to study the relationship between the dependent variable and one x-variable with the rest of the x-variables given fixed values
- Typically we are interested in the relationship x-y when the other variables are given values that
  - Maximizes y
  - Are averages values of of the x-variables
  - Minimizes y

Example based on the regression in table 3.2 in Hamilton

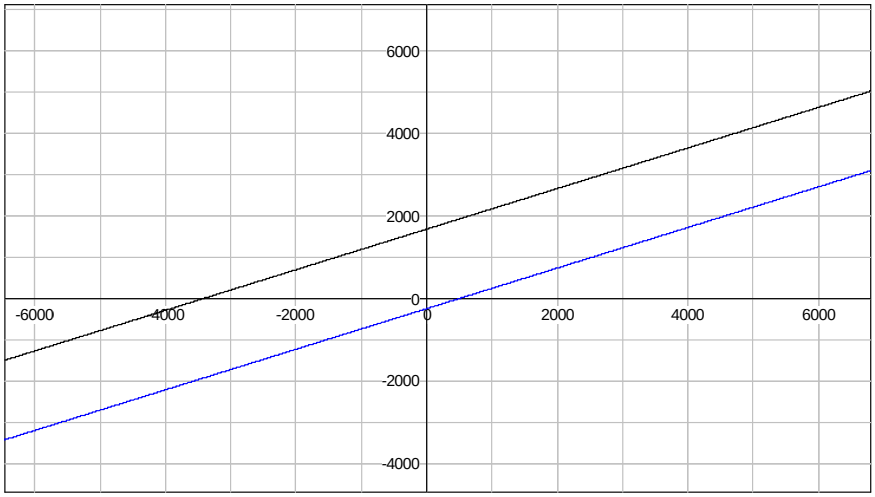
Dependent Variable: Summer 1981 Water Use	Unstandardized Coefficients			
	B	Std. Error	t	Sig.
(Constant)	242,220	206,864	1,171	,242
Summer 1980 Water Use	,492	,026	18,671	,000
Income in Thousands	20,967	3,464	6,053	,000
Education in Years	-41,866	13,220	-3,167	,002
head of house retired?	189,184	95,021	1,991	,047
# of People Resident, 1981	248,197	28,725	8,641	,000
Increase in # of People	96,454	80,519	1,198	,232

To produce conditional effect plots it is useful to have a table of minimum, maximum and average variable values

	N	Minimum	Maximum	Mean
Summer 1981 water use	496	100	10100	2298,39
Summer 1980 water use	496	200	12700	2732,06
Income in thousands	496	2	100	23,08
Education in years	496	6	20	14,00
Head of household retired?	496	0	1	,29
# of people resident, 1981	496	1	10	3,07
Relative increase in # of people	496	-3	3	-,04
# People living in 1980	496	1	10	3,11

The equation

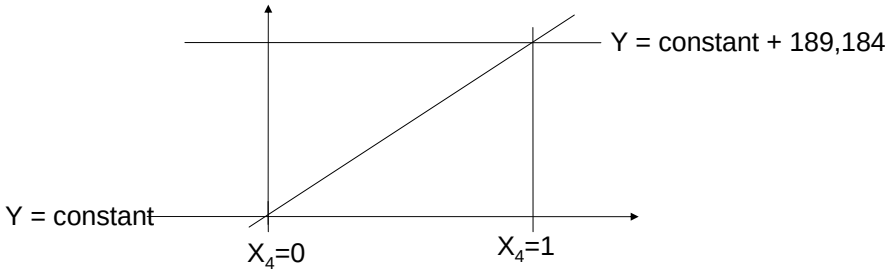
- Estimated Y = 242,22 + 0,492X_1 + 20,967X_2 - 41,866X_3 + 189,184X_4 + 248,197X_5 + 96,454X_6
- Maximizing the effect of X_1 on Y require maximum of X_2 , X_4 , X_5 , X_6 and minimum of X_3
- Average values of the effect of X_1 on Y is obtained by inserting average values of X_2 , X_3 , X_4 , X_5 , X_6
- Minimizing the effect of X_1 on Y require minimum of X_1 , X_2 , X_4 , X_5 , X_6 and maximum of X_3



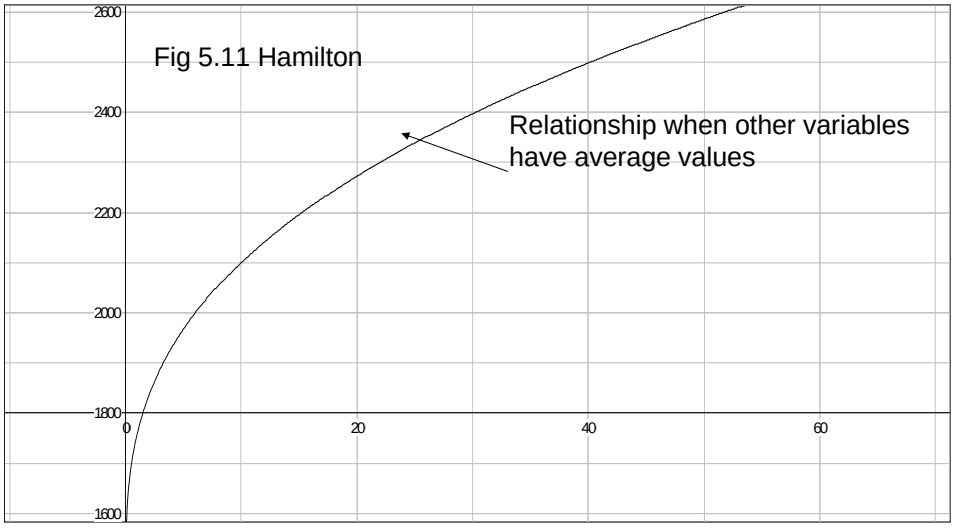
Y = 242.22 + 0.492X + 20.967×10 - 41.866×7 + 189.184×1 + 248.197×5 + 96.454×1
Y = 242.22 + 0.492X + 20.967×1 - 41.866×18 + 189.184×0 + 248.197×1 + 96.454×0

When x is dummy coded

- Estimated Y = 242,22 + 0,492X_1 + 20,967X_2 - 41,866X_3 + 189,184X_4 + 248,197X_5 + 96,454X_6
- Estimated Y = constant + 189,184X_4
  - X_4 can take the values of 0 or 1



Water usage according to income controlled for the effect of other variables



$y^{0.3}$=1.856+0.626(2732)$^{0.3}$-0.036(14)+0.101(0.294)+0.715ln(3.07)+0.916(ln(3.07)-ln(3.11))+0.516(x)$^{0.3}$

Which plots might be of interest?

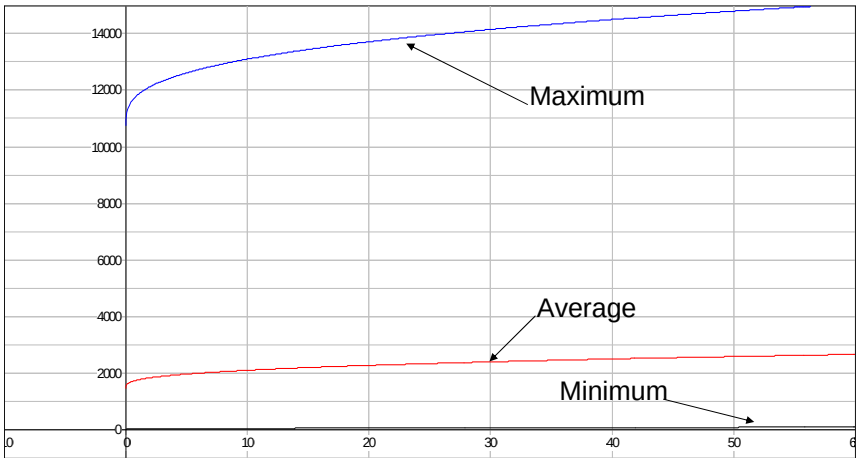
- The relationship between water usage and income controlled for the effect of other variables
  - Those minimizing water usage
  - Those maximizing water usage
  - Average values

1 $y^{0.3}=(1.856+0.626(200)^{0.3}-0.036(20)+0.101(0)+0.715\ln(1)+0.916(\ln(1)-\ln(10))+0.516(x)^{0.3})$

2 $y^{0.3}=(1.856+0.626(12700)^{0.3}-0.036(6)+0.101(1)+0.715\ln(10)+0.916(\ln(10)-\ln(1))+0.516(x)^{0.3})$

3 $y^{0.3}=(1.856+0.626(2732)^{0.3}-0.036(14)+0.101(0.29)+0.715\ln(3.07)+0.916(\ln(3.07)-\ln(3.11))+0.516(x)^{0.3})$

Comparing three types of usage



Relationship between water usage and income Fig 5.12 in Hamilton

The role of the constant in the plot

- The only difference between the three curves is the constant (konst)
  - In the maximum curve: (konst) = 14.046
  - In the minimum curve: (konst) = 4.204
  - In the average curve: (konst) = 8.507

$$y_i^{0.3} = \left(konst \right) + 0.516 x_{1i}^{0.3}$$

- The effect of income varies with the value of (konst)
- When we transform the dependent variable all relationships become interaction effects

Comparing effects

- For some relationships the standardized regression coefficient can be used to compare effects, but it is sensitive for biased estimates of the standard error
- A more general method is to compare conditional effect plots where the scaling of the y-axis is kept constant

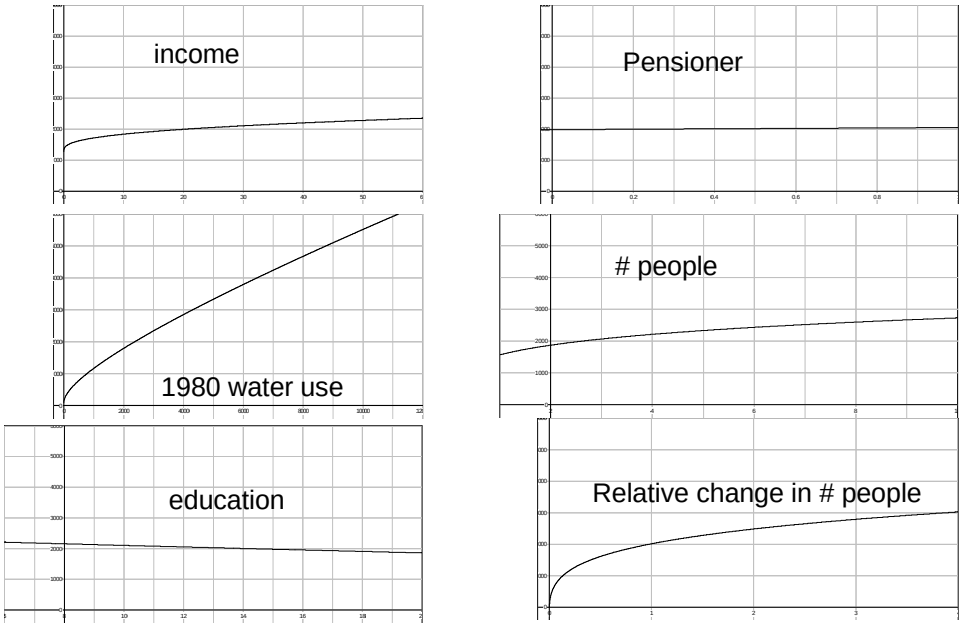
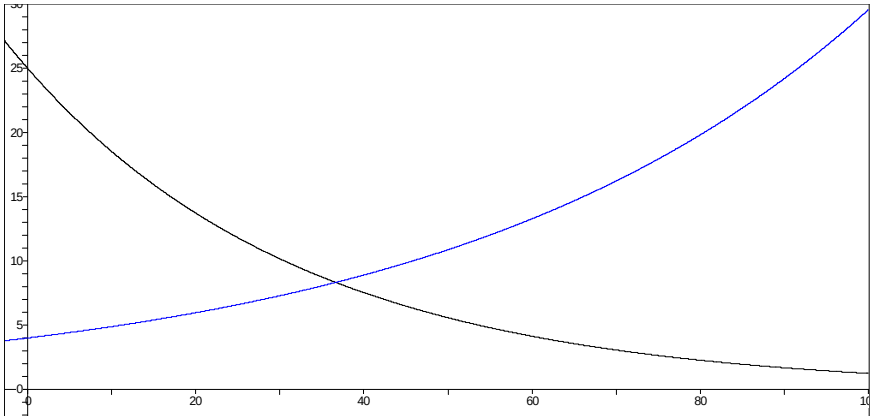


Fig 5.13 Hamilton

Non-linear models

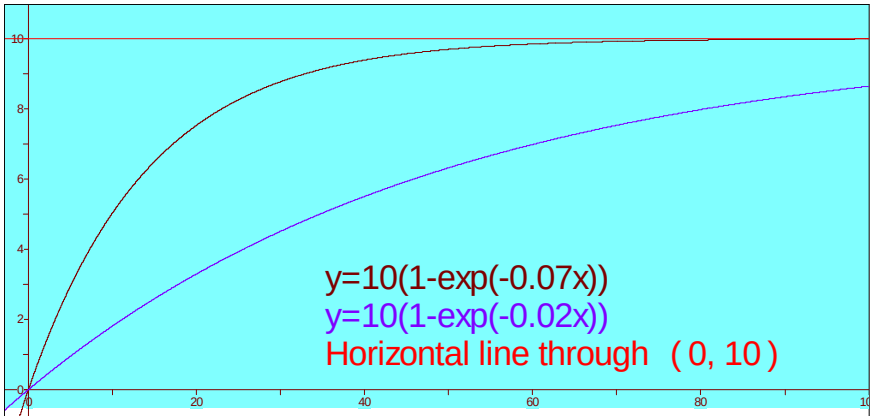
- If we do not have a model that is linear in the parameters other techniques than OLS are needed to estimate the parameters
- One may find two types of arguments for such models
  - Theory about the causal mechanism may say so
  - Inspection of the data may point towards one particular type of model
- We shall take a look at
  - Exponential models
  - Logistic models
  - Gompertz models

Exponential growth and decay
Fig 5.14 in Hamilton

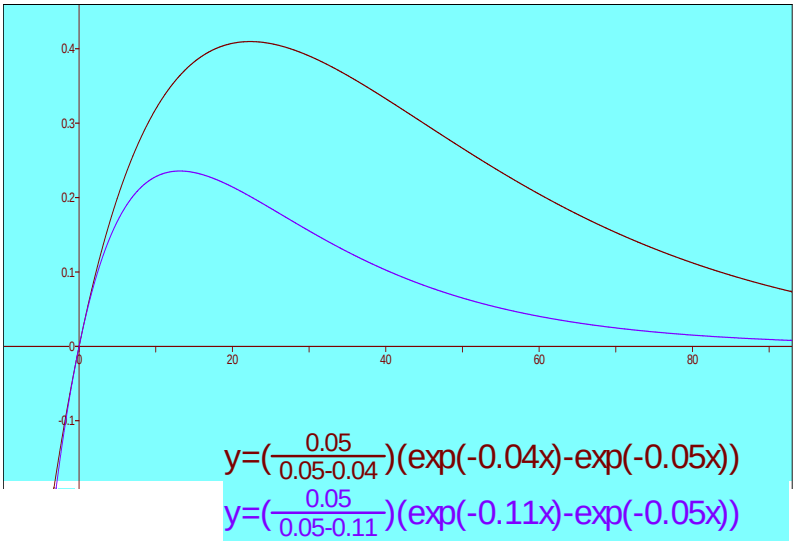


y=25exp(-0.03x)
y=4exp(0.02x)

Negative exponential curves Fig 5.15 in Hamilton



To-term exponential curves Fig 5.16 in Hamilton



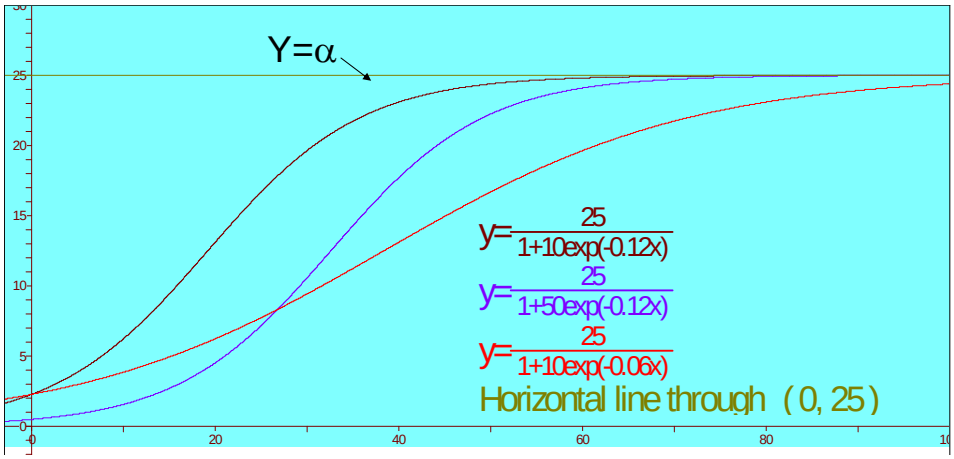
Logistic models

- The logistic function is written

$$y = \frac{\alpha}{1+\gamma \exp\left(-\beta x\right)}$$

- As x grows towards infinity y will approach α
- When x declines towards minus infinity y will approach 0
- Logistic models are appropriate for many phenomena
  - Growth of biological populations
  - Scattering of rumours
  - Distribution of illnesses

Logistic curves Fig 5.17 in Hamilton



- γ determines where growth starts
- β determines how fast the growth is

Logistic probability model

- If it is determined that α=γ=1 y will vary between 0 and 1 as x goes from minus infinity to plus infinity
- Logistic curves can then be used to model probabilities

$$y_i = \frac{1}{1+\exp\left(-\beta x_i\right)} + \varepsilon_\iota$$

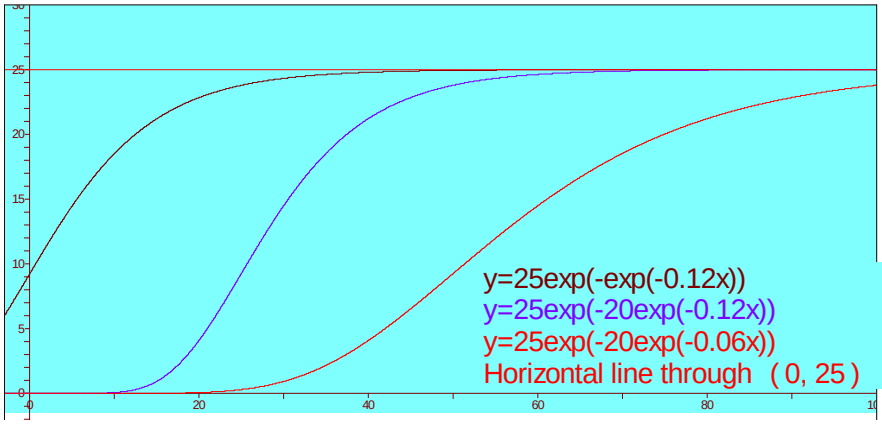
Gompertz curves

- Gompertz curves are sigmoid curves like the logistic, but growth increase and growth reduction occur at different rates. Hence they are not symmetric

$$y = \alpha e^{-\gamma e^{-\beta x}} + \varepsilon$$

- Parameters α, γ, and β have the same interpretation as in the logistic model

Gompertz curves Fig 5.18 Hamilton



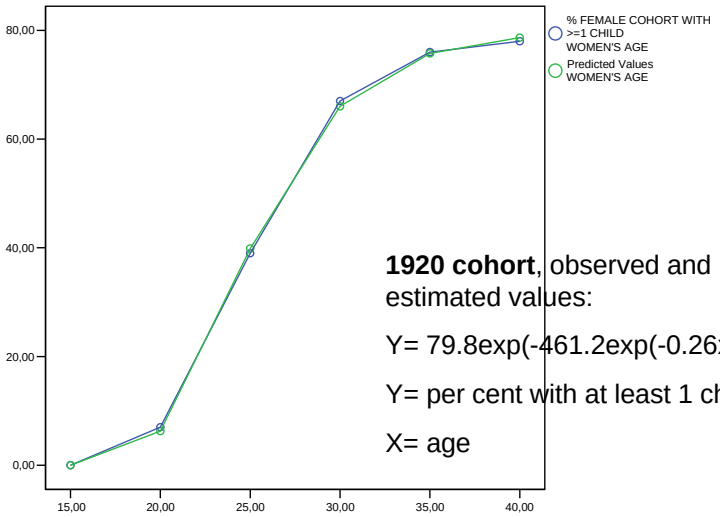
Estimation of non-linear models

- The criterion of fit is still minimum RSS
- It is uncommon to find analytical expressions for the parameters. One has to guess at a start value and go through several iterations to find which parameter value will give minimum RSS
- Good starting values are as a rule necessary, and everything from theory to inspection of data are used to find them

Per cent women with at least 1 child according to the woman's age and year of birth (England og Wales)

	1920	1930	1940	1945	1950	1955	1960	1965
15	0	0	0	0	0	0	0	0
20	7	9	13	17	19	18	13	11
25	39	48	59	60	53	45	39	-
30	67	75	82	82	75	68	-	-
35	76	83	87	88	83	-	-	-
40	78	86	89	90	-	-	-	-
45	-	86	89	-	-	-	-	-

Estimating Gompertz-models for cohorts (1)



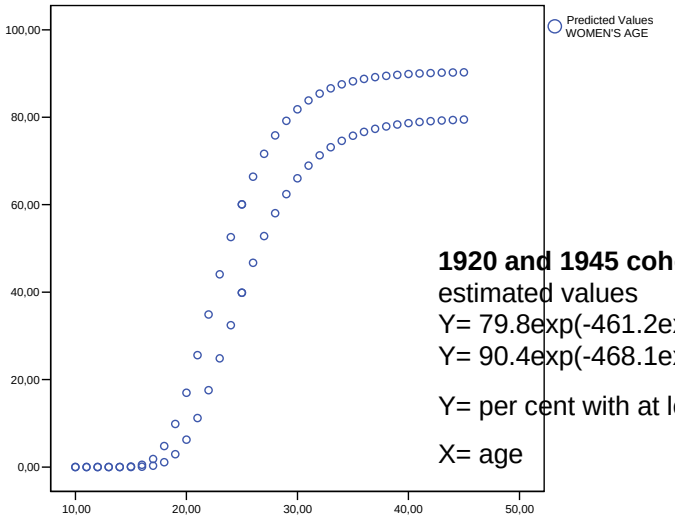
1920 cohort, observed and estimated values:

Y= 79.8exp(-461.2exp(-0.26x))

Y= per cent with at least 1 child

X= age

Estimating Gompertz-models for cohorts (2)



1920 and 1945 cohorts, estimated values

Y= 79.8exp(-461.2exp(-0.26x))

Y= 90.4exp(-468.1exp(-0.28x))

Y= per cent with at least 1 child

X= age

Model estimation and fit

- To evaluate a theoretically developed model
- To predict y within or outside the observed range of variation for x
- Substantial or comparative interpretation of the parameters of the model
  - On cohorts that are not finished with their births (thus predicting outside the observed range of x)
  - We can use the model to compare parameter values of different cohorts

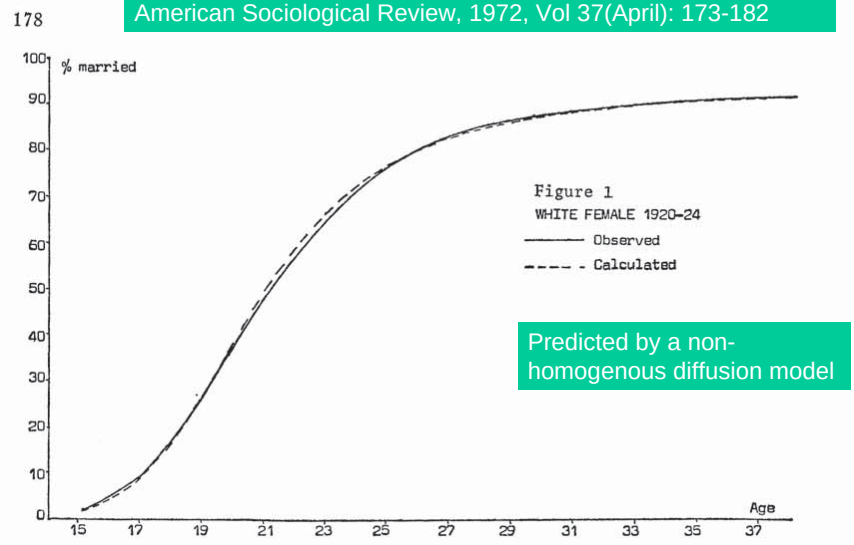
Parameter interpretation
Table 5.6 Hamilton

Cohort	α = upper limit	γ = ?	β = growth speed
1920	79.8	461.2	0.26
1930	86.5	538.0	0.27
1940	89.1	942.0	0.31
1945	90.4	468.1	0.28
1950	87.5	144.9	0.23
1955	88.9	60.3	0.18

The process of entry into first marriage

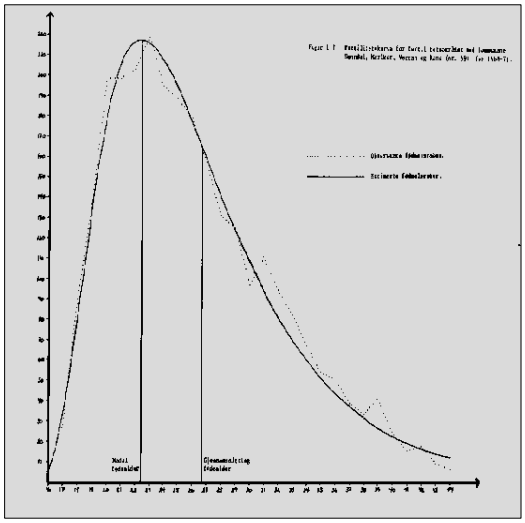
Gudmund Hernes

American Sociological Review, 1972, Vol 37(April): 173-182



Predicted by a non-homogenous diffusion model

Birth rates in Sunndal, Meråker, Verran, and Rana 1968-71



- Estimated with a Hadwiger function
- Ref.: Berge, Erling. 1981. The Social Ecology of Human Fertility in Norway 1970. Ph.D. Dissertation. Boston: Boston University.

Conclusions of chapter 5 (1)

- Data analysis often starts with linear models. They are the simplest.
- Theory or exploratory data analysis (band regression, smoothing) can tell us if curvilinear or non-linear models are needed
- Transformation of variables give curvilinear regression. This can counteract several problems:
  - Curvilinear relationships
  - Case with large influence
  - Non-normal errors
  - Heteroscedasticity

Conclusions of chapter 5 (2)

- Non-linear regression use iterative procedures to find parameter estimates
- The procedures need initial values and are often sensitive for the initial values
- The interpretation of the parameters may be difficult. Graphs showing the relationship for different parameter values will provide valuable help for the interpretation

Ch 6 Robust Regression

- Has been developed to work well in situations where OLS breaks down. Where the OLS assumptions are satisfied robust regression are not as good as OLS, but not by very much
- Even if robust regression is better suited for those who do not want to put much effort into testing the assumptions, it is so far difficult to use
- Robust regression has focused on residuals with heavy tails (many cases with high influence on the regression)

Regression of mortality on air pollution

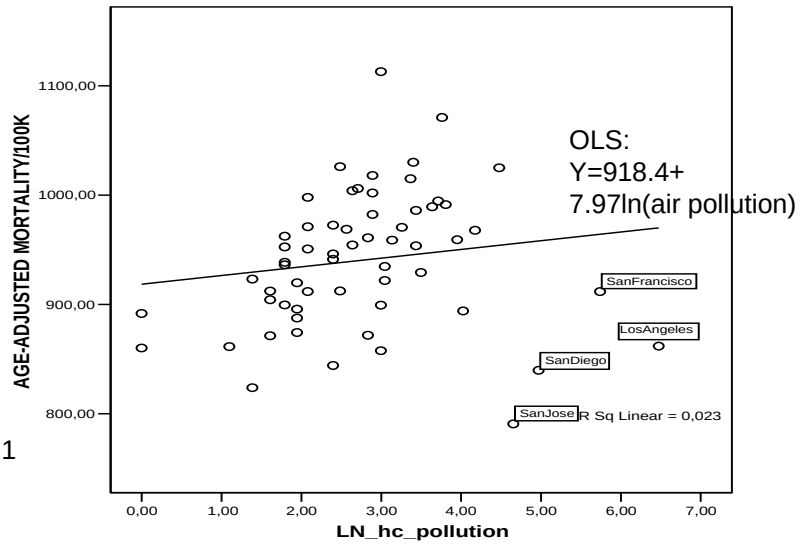


Figure 6.1
Hamilton

Robust regression of mortality on air pollution

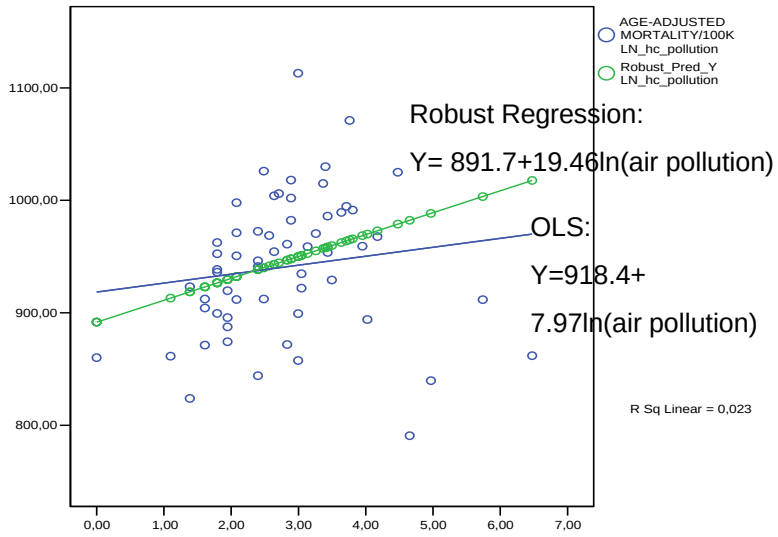


Figure 6.2
Hamilton

Robust regression and SPSS

- SPSS do not have a particular routine that performs robust regression
- It can possibly be done within the Generalized linear models procedure <but I have not tested it>
- It can be done by weighted OLS regression, but then it is required that we make the weight functions and go through the iterations one by one including computation of weights every time
- This procedure will be outlined below

ROBUST AND RESISTANT

- RESISTANT methods are not affected by small errors or changes in the sample data
- ROBUST methods are not affected by small deviations from the assumptions of the model
- Most resistant estimators are also robust in relation to the assumption about normally distributed residuals
-
- **OLS is neither ROBUST nor RESISTANT**

Outliers is a problem for OLS

Outliers affect the estimates of

- Parameters
- Standard errors (standard deviation of parameters)
- Coefficient of determination
- Test statistics
- And many other statistics

Robust regression tries to protect against this
by giving less weight to such cases,
not by excluding them

Protection against NON-NORMALE residuals

Robust methods can help when

- the tails in the distribution of the residuals are heavy, i.e. when it is too many outliers compared to the normal distribution
- Unusual X-values have leverage and may cause problems

But for other causes of non-normality

robust methods will not help

Estimation methods for robust regression

- M-estimation (maximum likelihood) minimizes a weighted sum of the residuals. This can be approximated by the weighted least squares method (WLS)
- R-estimation (based on rank) minimizes a sum where a weighted rank is included. The method is more difficult to use than M-estimation
- L-estimation (based on quantiles) uses linear functions of the sample order statistics (quantiles)

IRLS-
Iterated Reweighted Least Squares

M-estimation by means of IRLS needs

1. Start values from OLS. Save the residuals
2. Use OLS residuals to find weights. Larger residuals gives less weight
3. Find new parameter values and residuals with WLS
4. Go to step 2 and find new weights from the new residuals, go on to step 3 and 4, until changes in the parameters become small

Iteration: to repeat a sequence of operations

IRLS

- IRLS is in theory equivalent to M-estimation
- To use the method we need to compute
- Scaled residuals, u_i , and a
- Weight function, w_i ,that gives least weight to the largest residuals

Scaling of residuals I

- Scaled residual u_i
  - s is the scale factor and e_i residual
- The scale factor in OLS is the estimate of the standard error of the residual: nb! s_e is not resistant
- A resistant alternative is based on MAD, "median absolute deviation"

$$u_i = \frac{e_i}{s}$$

$$s_e = \sqrt{\frac{RSS}{n-K}}$$

$$MAD = median\left|e_i - median\left(e_i\right)\right|$$

Scaling of residuals II

$$MAD = median\left|e_i - median\left(e_i\right)\right|$$

The scale factor (standard error of the distribution)
Using a resistant estimate will be

- s = MAD/ 0.6745 = 1.483MAD

and the scaled residual

- $u_i = [e_i / s] = (0.6745*e_i)/MAD$

In a normal distribution s= MAD/ 0.6745 will estimate the standard error correctly like s_e
In case of non-normal errors s= MAD/ 0.6745 will be better.
This is a resistant estimate, s_e is not resistant

Weight functions I

- Properties is measured in relation to OLS on normally distributed errors.
- The method should be “almost as good” as OLS on normally distributed errors and much better when the errors are non-normal
- Properties are determined by a “calibration constant” (c in the formulas)

Weight functions II

- **OLS-weights**: $w_i = 1$ for all i
- **Huber-weights**: weights down when the scaled residual is larger than c, c=1,345 gives 95% of the efficiency of OLS on normally distributed errors
- **Tukey’s bi-weighted** estimates get 95% of the efficiency of OLS on normally distributed errors by gradually weighting down scaled errors until $|u_i| \leq c = 4.685$ and by dropping cases where the residual is larger

Huber-weights

$$w_i = 1 \; \forall \mid \mathrm{u_i} \mid \leq \mathrm{c}$$

$$w_i = \frac{c}{u_i} \forall \mid \mathrm{u_i} \mid > \mathrm{c}$$

$$\forall = \text{for alle}$$

Tukey weights

$$w_i = \left[1 - \left(\frac{u_i}{c} \right)^2 \right]^2 \; \forall \mid \mathrm{u_i} \mid \leq c$$

$$w_i = 0 \; \forall \mid \mathrm{u_i} \mid > c$$

$$\forall = \textit{for alle}$$

- Tukey weighting in IRLS is sensitive for start values of the parameters (one may end up at local minima)

Standard errors and tests in IRLS

- The WLS program cannot estimate standard errors and test statistics correctly by IRLS
- A procedure that works is described by Hamilton on page 198-199

Use of Robust Estimation

- If OLS and Robust estimates are different it means that outliers have influence on the OLS results making them unreliable. Results cannot be trusted
- Robust predicted values will better portray the bulk of the data
- Robust residuals will be better at discovering which cases are unusual
- Weights from the robust regression will show which cases are outliers
- OLS and RR can support each other

Fig 6.9 Hamilton: OLS and RR on untransformed data

Mortality regressed on air pollution

Effect of high leverage

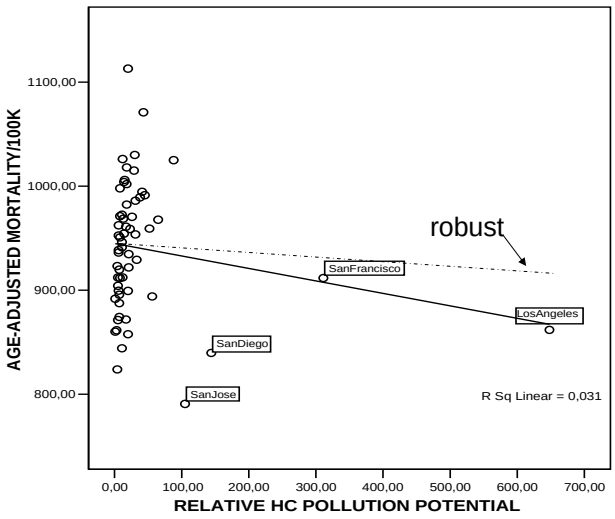
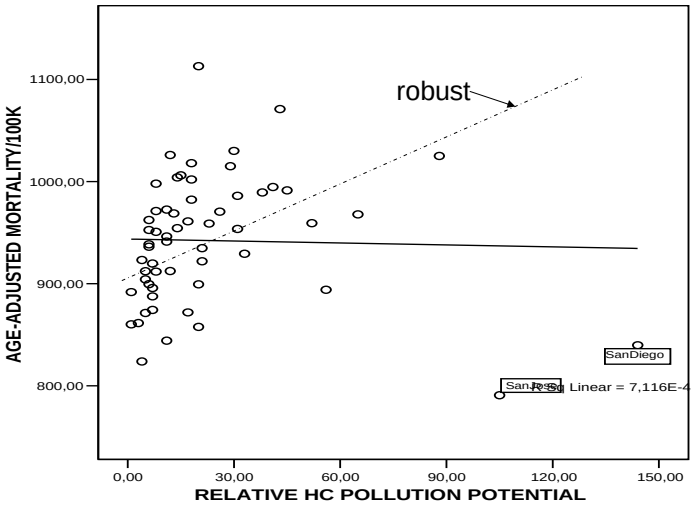


Fig 6.10 Hamilton: OLS and RR on untransformed data when two outliers are removed

Mortality regressed on air pollution



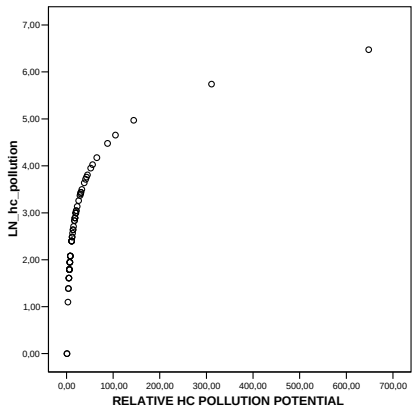
RR do not protect against leverage

- RR with M-estimation protects against unusual y-values (outliers) but not necessarily against unusual x-values (leverage)
- Efforts to test and diagnose are still needed (heteroscedasticity is still a problem for IRLS)
- Studies of the data and transformation to symmetry will reduce the risk of problems appearing
- No method is “safe” if it is used without forethought and diagnostic studies of data

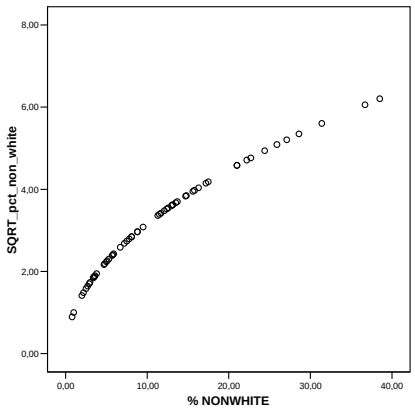
Robust Multippel Regresjon

X_1	RELATIVE HC POLLUTION POTENTIAL (natural log)
X_2	AVG. YEARLY PRECIP. INCHES
X_3	AVG. JANUARY TEMPERATURE, F
X_4	MEDIAN EDUCATION OF POP 25+
X_5	% NON-WHITE (square root)
X_6	POPULATION PER HOUSEHOLD
X_7	% 65 AND OVER
X_8	% SOUND HOUSING UNITS
X_9	PEOPLE PER SQUARE MILE (natural log)
X_{10}	AVG. JULY TEMPERATURE, F
X_{11}	% WHITE COLLAR EMPLOYMENT
X_{12}	% FAMILIES WITH INCOME<$3000 (negative reciprocal root)
X_{13}	AVG RELATIVE HUMIDITY, %

Multiple OLS regression with transformed variables: effect of transformation



ln of air pollution



Square root of % non-white

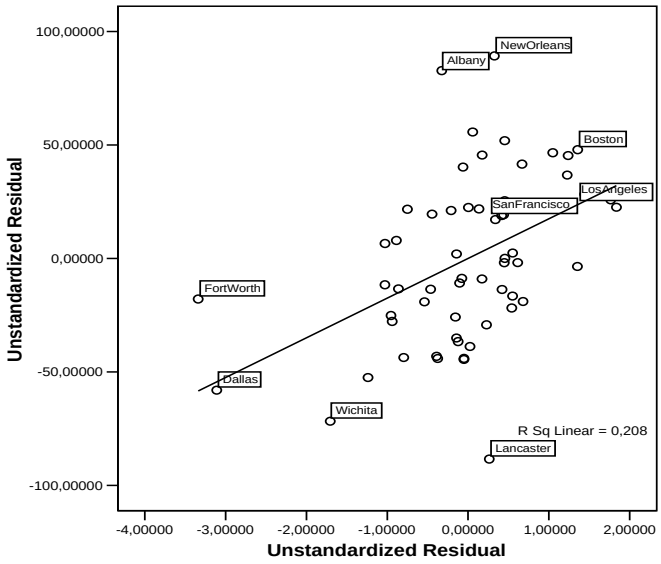
OLS with backward elimination gives

Dependent Variable: AGE-ADJUSTED MORTALITY/100K	B	Std. Error	t	Sig.
(Constant)	986,261	82,674	11,929	,000
LN_hc_pollution	17,469	4,636	3,768	,000
AVG. YEARLY PRECIP. INCHES	2,352	,640	3,677	,001
AVG. JANUARY TEMPERATURE, F	-2,132	,504	-4,228	,000
MEDIAN EDUCATION OF POP 25+	-17,958	6,204	-2,895	,005
SQRT_pct_non_white	27,335	4,398	6,215	,000

- Robust regression gives predicted y:
- $Y = 1001.8+17.77x_{1i}+2.32x_{2i}-2.11x_{3i}-19.1x_{4i}+26.2x_{5i}$

Multiple OLS regression with transformed variables

Leverage plot of residual from mortality (y) and residual of ln_air_pollution (x)



Los Angeles and San Francisco are no longer outliers

Four estimates of the relationship mortality – air pollution

Effect of air pollution

	OLS	Robust
1 variable	7.97	19.46
5 variables	17.47	17.77

- Note that in RR the bivariate regression comes pretty close to the result of the multivariate regression
- In the five-variable model there are new cases with influence on the line of regression
- Removing the 5 cases that have the highest leverage parameter (h_i) do not give substantial changes in the coefficients

Robust Regression vs Bounded Influence Regression

- Robust Regression protect against the effect of outliers (unusual y-values) if these do not go together with unusual x-values
- Bounded Influence Regression is designed to protect against influence from unusual combinations of x-values

BI - Bounded Influence Regression

- BI-methods are made to limit the influence of high leverage cases (large h_i = high leverage)
- The simplest way of doing this is to modify the Huber-weights or Tukey-weights in the IRLS procedure for RR (robust regression) with a factor based on the leverage statistic

Bounded influence: modification of weights

- Expand the weight function with a weight based on the leverage statistic h_i
- $w^H_i = 1$ if $h_i \leq c^H$
- $w^H_i = (c^H / h_i)$ if $h_i > c^H$
- c^H is often set to the 90% percentile in the distribution of h_i
- Then the IRSL weight becomes $w_i w^H_i$ where w_i is either the Tukey- or Huber-weight that changes from iteration to iteration while w^H_i is constant

Bounded influence as a diagnostic tool

- Estimation of standard errors and test statistics becomes even more complicated than for the M-estimators mentioned above
- We can use BI estimates as a descriptive tool to check up on other estimates
- One (somewhat) extreme example: PCB pollution in river mouths in 1984 and 1985 (Hamilton table 6.4)

Fig 6.15 and 6.16 Hamilton

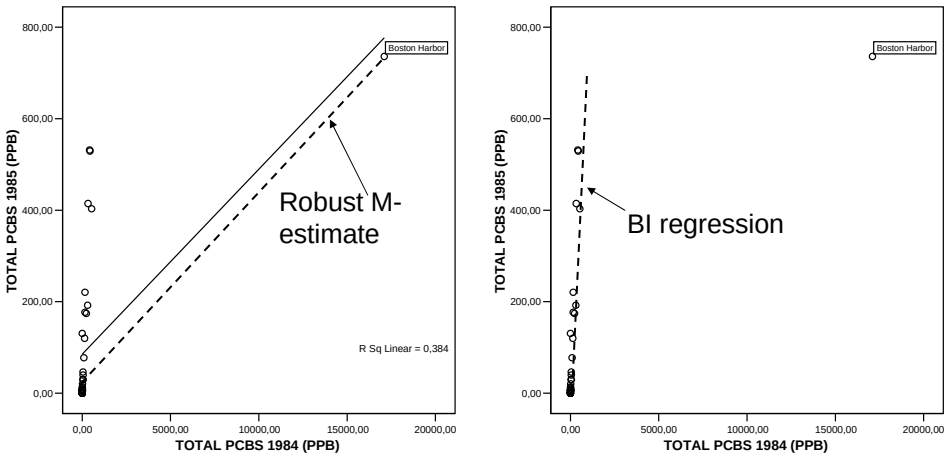
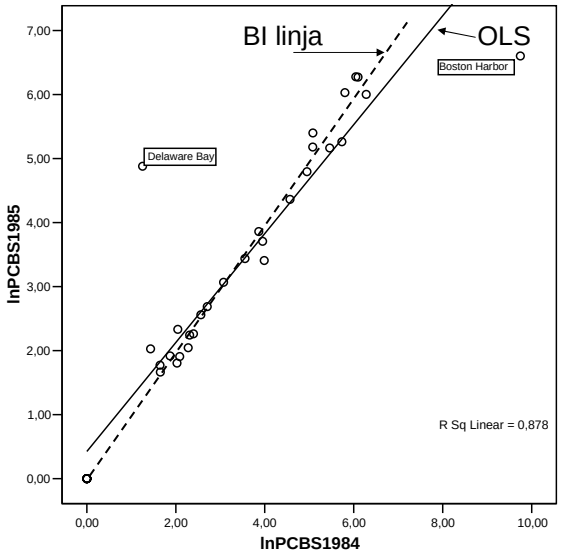


Fig 6.17 Hamilton

OLS and BI estimates with transformed variables give about the same result



Conclusions

- When data have many outliers robust methods will have better properties than OLS
  - They are more effective and give more accurate confidence intervals and tests of significance
- Robust regression can be used as a diagnostic tool
  - If OLS and RR agree we can have more confidence in the OLS results
  - If they disagree we will
    - Know that a problem exist
    - Have a model that fits the data better and identifies the outliers better
- Robust methods does not protect against problems that are due to curvilinear or non-linear models, heteroscedasticity, and autocorrelation